

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2024
Graph Theory

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

| Module – 1 | | | | M | L | C |
|---------------------------------|----|--|--|----|----|-----|
| Q.1 | a. | Define graph. List and explain the types of graph. | | 08 | L1 | CO1 |
| | b. | Prove that the number of vertices of odd degree in a graph is always even. | | 06 | L2 | CO1 |
| | c. | Define isomorphic graph and verify the following graphs are isomorphic or not. [Refer Fig.Q1(c)] | | 06 | L2 | CO1 |
| <p align="center">Fig.Q1(c)</p> | | | | | | |
| OR | | | | | | |
| Q.2 | a. | Explain the following graphs: (i) Bi-partite graph (ii) Sub graphs (iii) WALK (iv) Path | | 10 | L1 | CO1 |
| | b. | Prove that a simple graph with n vertices and K components can have at most $(n - K)(n - K + 1)/2$ edges. | | 10 | L2 | CO1 |
| Module – 2 | | | | | | |
| Q.3 | a. | State and prove necessary condition of a graph to be a Euler graph. | | 10 | L2 | CO2 |
| | b. | List and explain the different operations on graph. | | 10 | L2 | CO2 |
| OR | | | | | | |
| Q.4 | a. | Define digraph. Find the indegree and outdegree of the following graph [Fig.Q4(a)]. | | 08 | L2 | CO2 |
| | | <p align="center">Fig.Q4(a)</p> | | | | |
| | b. | Illustrate the travelling salesman problem using a graph. | | 06 | L2 | CO2 |
| | c. | List and explain different digraphs and binary relations. | | 06 | L2 | CO2 |
| Module – 3 | | | | | | |
| Q.5 | a. | Define a tree. Prove that in a graph G there is one and only one path between every pair of vertices, G is a tree. | | 06 | L1 | CO3 |

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|-------------------|----|---|----|----|-----|
| | b. | Explain the following: (i) Cut-edge (ii) Cut-vertex (iii) Cut-set | 06 | L1 | CO3 |
| | c. | Find and construct the following: (i) Minimum possible height of 11 vertex binary tree (ii) A binary tree for a given 11 such that the farthest vertex is as far as possible from the root that must have exactly 2 vertices at each level, except at zero level. | 08 | L2 | CO3 |
| OR | | | | | |
| Q.6 | a. | Prove that every circuit has an even number of edges in common with any cut set. | 10 | L2 | CO3 |
| | b. | Prove that ring 50 m of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets. | 10 | L2 | CO3 |
| Module – 4 | | | | | |
| Q.7 | a. | Define the following: (i) Planar graph (ii) Embedding (iii) Non-planar (iv) Kuratowski's 2 graph | 08 | L2 | CO4 |
| | b. | Explain the simple observation mode relationship between planar graph and dual G^* . | 08 | L2 | CO4 |
| | c. | Write a note on path matrix. | 04 | L1 | CO4 |
| OR | | | | | |
| Q.8 | a. | Prove that two graphs G_1 and G_2 are isomorphic if and only if their incidence matrices $A(G_1)$ and $A(G_2)$ differ only by permutations of rows and columns. | 10 | L2 | CO5 |
| | b. | Describe the observations that can be made about circuit matrix $B(G)$ of graph G . | 10 | L2 | CO5 |
| Module – 5 | | | | | |
| Q.9 | a. | Prove that every tree with two or more vertices is 2 - chromatic. | 10 | L2 | CO5 |
| | b. | Explain the following for chromatic polynomial: (i) Finding a maximal independent set (ii) Finding all maximal independent set. | 10 | L2 | CO5 |
| OR | | | | | |
| Q.10 | a. | Prove that the vertices of every planar graph can be properly colored with five colors. | 10 | L2 | CO5 |
| | b. | Explain the Greedy colouring algorithm. | 10 | L2 | CO5 |

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