CBCS SCHEME

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18CS42

Fourth Semester B.E. Degree Examination, June/July 2024
Design and Analysis of Algorithms

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define an algorithm. Design an algorithm to search an element in an array of elements using sequential search. Discuss worst, best and average efficiency of the algorithm. **(08 Marks)**

b. Explain the asymptotic notations: i) Big-oh (O) ii) Big omega (Ω) iii) Theta (θ) with an example for each. Also prove that: If $t_1(n) \in O(g_1(n))$ and $t_2(n) \in O(g_2(n))$ then $t_1(n) + t_2(n) \in O(\max\{g_1(n), g_2(n)\})$. **(08 Marks)**

c. Discuss adjacency matrix and adjacency list representation of a graph with suitable examples. **(04 Marks)**

OR

2 a. Write a recursive algorithm to print all the permutations of a set of $n \geq 1$ elements. Also write a recursive tree of calls for $n = 3$. **(08 Marks)**

b. Give a general plan for analyzing recursive algorithms. Give a recursive algorithm to find the number of binary digits in the binary representation of a positive decimal integer and obtain its efficiency. **(08 Marks)**

c. Explain with an example, how to convert ordered rooted tree into a binary tree. **(04 Marks)**

Module-2

3 a. Given a bag of 16 coins and let seventh one be the counterfeit coin among 16 coins. Counterfeit coins are lighter than genuine ones. Given a machine to compare the weights of two sets of coins, apply divide and conquer to determine that 7^{th} is counterfeit **(06 Marks)**

b. Apply merge sort to sort the following numbers in ascending order: 8, 3, 2, 9, 7, 1, 5, 4. Also obtain time complexity for merge sort. **(08 Marks)**

c. Apply divide and conquer approach to multiply the following matrices:

$$\begin{bmatrix} 5 & 2 & 6 & 1 \\ 0 & 6 & 2 & 0 \\ 3 & 8 & 1 & 4 \\ 1 & 8 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 5 & 8 & 0 \\ 1 & 8 & 2 & 6 \\ 9 & 4 & 3 & 8 \\ 5 & 3 & 7 & 9 \end{bmatrix}$$ **(06 Marks)**

OR

4 a. Write an iterative algorithm for binary search and trace the algorithm to search for a key 151 in a list -15, -6, 0, 7, 9, 23, 54, 82, 101, 112, 125, 131, 142, 151. **(06 Marks)**

b. Write quicksort algorithm to sort 'n' numbers and apply the same to sort the following numbers in ascending order. 80, 60, 70, 40, 10, 30, 50, 20. **(08 Marks)**

c. Apply DFS and source vertex removal methods to obtain topological sequence for two graph shown in Fig.Q.4(c) **(06 Marks)**

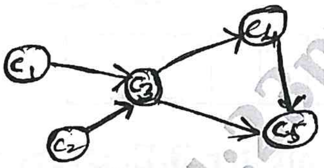

Fig.Q.4(c)

Module-3

5 a. Apply greedy method to obtain an optimal task assignment for the 7 tasks and infinite supply of machines given their start end times.

Task	a	b	c	d	e	f	g
Start time	0	3	4	9	7	1	6
End time	2	7	7	11	10	5	8

(06 Marks)

b. Solve the following knapsack problem using all 3 greedy criteria: m = 20, n = 3, p = (25, 24, 15) w = (18, 15, 10). **(06 Marks)**

c. Write and apply kruskals algorithm to obtain minimum cost spanning tree for the graph shown in Fig.Q.5(c). **(08 Marks)**

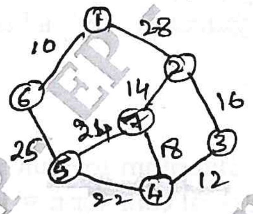

Fig.Q.5(c)

OR

6 a. Write and apply prims algorithm to obtain minimum cost spanning tree for the graph shown in Fig.Q.5(c). **(08 Marks)**

b. Apply Dijkstra algorithm to find the shortest distance from vertex 1 to all nodes in a graph shown in Fig.Q.6(b) **(06 Marks)**

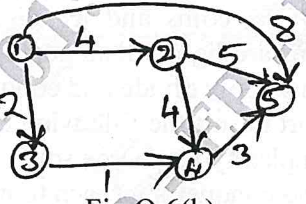

Fig.Q.6(b)

c. Determine the optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies: 20, 28, 4, 17, 12, 7 respectively. **(06 Marks)**

Module-4

7 a. Define multistage graph problem and write the forward approach algorithm to obtain a solution. **(06 Marks)**

b. Write Floyd's algorithm to solve all pairs shortest path problem and apply the same for the graph shown in Fig.Q.7(b). **(08 Marks)**

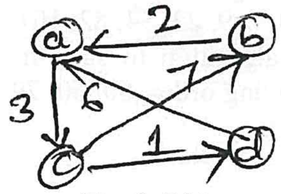

Fig.Q.7(b)

c. Find optimal tour for the travelling sales person problem using dynamic programming technique for the graph shown in Fig.Q.7(c) considering initial and end vertex as 1. **(06 Marks)**

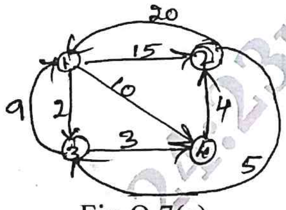

Fig.Q.7(c)

OR

8 a. Define transitive closure of a digraph. Find the transitive closure matrix for the graph whose adjacency matrix is $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ **(08 Marks)**

b. Write the algorithm to construct optimal binary search tree for the following data:

Key	A	B	C	D
Probability	0.1	0.2	0.4	0.3

(12 Marks)

Module-5

9 a. Define n-queens problem. Construct state-space tree for solving 4-queens problem using backtracking for all possible solutions. **(08 Marks)**

b. Solve the following assignment problem using branch and bound technique, whose cost matrix for assigning four jobs to four persons are given.

$$\begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

(06 Marks)

c. Explain the classes of NP-hard and NP-complete. **(06 Marks)**

OR

10 a. State the subset sum problem. Using backtracking, obtain a solution to the subset sum problem given s = {6, 8, 2, 14} and d = 16. **(06 Marks)**

b. Construct a space tree representing all possible colorings using atmost 3 colors for the graph shown in Fig.Q.10(b). **(07 Marks)**

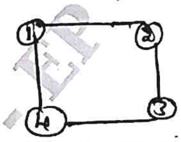

Fig.Q.10(b)

c. With the help of a state-space tree, solve the travelling sales person problem Fig.Q.10(c) using branch-and-bound technique. **(07 Marks)**

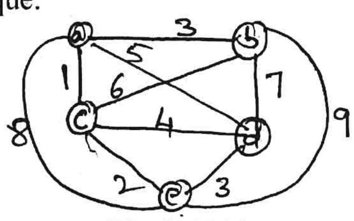

Fig.Q.10(c)

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