CBCS SCHEME

USN | | | | | | | | | |

18CS36

Third Semester B.E. Degree Examination, June/July 2024
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Define the following :
(i) Proposition
(ii) Tautology
(iii) Contradiction
Verify whether the following compound proposition is a Tautology
$[(p \to q) \wedge (q \to r)] \to (p \to r)$ **(07 Marks)**

b. By constructing the Truth Table show that,
$[(p \vee q) \to r] \Leftrightarrow [(p \to r) \wedge (q \to r)]$ **(07 Marks)**

c. Let $p(x): x^2 - 7x + 10 = 0$, $q(x): x^2 - 2x + 3 = 0$, $r(x): x < 0$
Find the truth or falsity of the following statements, when the universe U contains only the integers 2 and 5.
(i) $\forall x, p(x) \to \neg r(x)$ (ii) $\forall x, q(x) \to r(x)$ (iii) $\exists x\, q(x) \to r(x)$
(iv) $\exists x\, p(x) \to r(x)$ **(06 Marks)**

OR

2 a. Define Converse, Inverse and Contra-positive of a conditional. Represent these in the form of a truth table. **(07 Marks)**

b. For each of the following statements, provide an indirect proof by stating and proving the contrapositive of the given statement.
(i) For all integers k and ℓ, if k ℓ is odd, then both k and ℓ are odd.
(ii) For all integers k and ℓ, if k+ ℓ is even, then k and ℓ are both even and odd. **(07 Marks)**

c. (i) I will get grade A in this coarse or I will not graduate.
If I do not graduate, I will join the army
I got grade A.

$\therefore$ I will not join the army.
Is this a valid argument?

(ii) If Ravi studies, he will pass in Discrete Mathematics paper
If Ravi does not paly PUBG, then he will study,
Ravi failed in Discrete Mathematics paper

$\therefore$ Ravi played PUBG
Check the validity of this argument. **(06 Marks)**

Module-2

3 a. Prove by mathematical induction that, for every positive integer n, 5 divides $n^5 - n$. **(06 Marks)**

b. For all positive integers n, prove that, if $n \geq 24$, then n can be written as a sum of 5's and 7's. **(07 Marks)**

c. Define the well ordering principle, by using mathematical induction principle, prove that $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{0.11} + ... + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$. **(07 Marks)**

OR

4 a. Prove by mathematical induction that, for every positive integer n, the number $11^{n+2} + 12^{2n+1}$ is divisible by 133. **(06 Marks)**

b. Determine the coefficient of,

(i) $x^{11}y^4$ in the expansion of $(2x^3 - 3xy^2 + z^2)^6$.

(ii) $a^2b^3c^2d^5$ in the expansion of, $(a + 2b - 3c + 2d + 5)^{16}$ **(07 Marks)**

c. In how many ways can one distribute eight identical balls into four distinct containers so that,

(i) No container is empty?

(ii) The fourth container gets an odd number of balls? **(07 Marks)**

Module-3

5 a. Let $f: R \rightarrow R$ be defined by $f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$.

(i) Determine f(0), f(-1), $f\left(\frac{5}{3}\right), f\left(\frac{-5}{3}\right)$.

(ii) Find $f^{-1}[(-5,5)]$ **(07 Marks)**

b. Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$, $C = \{3, 4, 7\}$. Write down the following $A \cup (B \times C)$, $(A \cup B) \times C$, $(A \times C) \cup B \times C$. **(07 Marks)**

c. Let f, g, h be functions from Z to Z defined by $f(x) = x - 1$, $g(x) = 3x$,

$h(x) = 0$ if x is even

$= 1$ if x is odd.

Determine $(f \circ (g \circ h))(x) = ((f \circ g) \circ h)(x)$ and verify $(f \circ (g \circ h)) = ((f \circ g) \circ h)$. **(06 Marks)**

OR

6 a. State pigeonhole principle. Let ΔABC be an equilateral triangle of side 1 cm each. Show that if we select 10 points in the interior, there must be at least two points whose distance is less than $\frac{1}{2}$ cm. **(07 Marks)**

b. Draw Hasse diagram representing the positive divisors of 36. **(07 Marks)**

c. Let $A = \{1, 2, 3\}$ and f, g, h, p be functions on A defined as follows :

$f = \{(1,2)(2,3)(3,1)\}$, $g = \{(1,2)(2,1)(3,3)\}$, $h = \{(1,1)(2,2)(3,1)\}$, $p = \{(1,1)(2,2)(3,2)\}$

Find $f \circ g$, $g \circ f$, $f \circ p$, $p \circ g$, $g \circ p$, $f \circ h \circ g$ **(06 Marks)**

Module-4

7 a. Define derangement. **(07 Marks)**

(i) There are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that at least one letter goes to the right person.

(ii) Find the number of derangement of 1, 2, 3, 4.

b. Find the rook polynomial for the 3×3 board by using the expansion formula. **(07 Marks)**

c. Define homogeneous and non-homogeneous recurrence relations of first order and solve the recurrence relation.

$a_n = 5a_{n-1} + 6a_{n-2}$, $n \geq 2$, $a_0 = 1$, $a_1 = 5$. **(06 Marks)**

OR

8 a. Solve the following :

(i) From seven consonants and five vowels how many sets consisting of four different consonants and three different vowels can be formed?

(ii) Find the number of arrangement of the letters in TALLAHASSE. Which have no adjacent A's. **(07 Marks)**

b. How many integers between 1 and 300 (inclusive) are,

(i) Divisible by 5, 6, 8?

(ii) Divisible by none of 5, 6, 8? **(07 Marks)**

c. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3 and B_4. The boys B_1 and B_2 do not wish to have apple, the boy B_3 does not want banana or mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(06 Marks)**

Module-5

9 a. Define the following terms with respect to graph :

(i) Directed graph
(ii) A walk
(iii) Sub-graph
(iv) Connected graph
(v) Simple graph
(vi) Regular graph
(vii) Complete graph **(07 Marks)**

b. Define complete bipartite graph. How many vertices and how many edges are there in $K_{4,7}$ and K_{7111}? **(07 Marks)**

c. Let $G=(V,E)$ be the undirected graph in the Fig. Q9 (c). How many paths are there in G from a to h? How many of these paths have a length 5? **(06 Marks)**

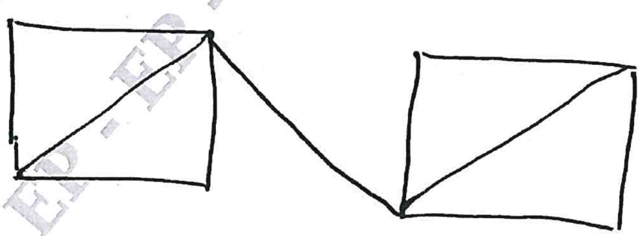

Fig. Q9 (c)

OR

10 a. Define a Tree. Prove that the tree G = (V, E) with P vertices has (P – 1) edges. **(07 Marks)**

b. Define a spanning tree of a graph. Find all the spanning trees of the following graph in Fig.Q10 (b). **(07 Marks)**

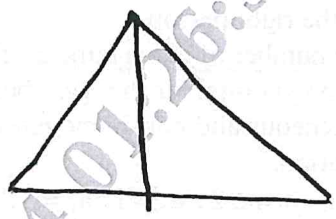

Fig. Q10 (b)

c. Obtain an optimal prefix code for the message LETTER RECEIVED using labelled binary tree. Indicate the code. **(06 Marks)**

* * * * *